

Elementary Algebra

REVIEW

Lenoir Community College, Kinston, NC



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EQUATIONS AND INEQUALITIES

Simplifying Algebraic Expressions

Simplifying generally means creating equivalent expressions that contain fewer additions and or multiplications. To simplify algebraic expressions, combine like terms by adding their coefficients and keeping the variable parts the same. To simplify expressions containing parentheses, remove all parentheses and combine like terms. Remember to work problems containing parentheses from the inner most set outward.

Example:	$-(X-2Y) - 3(4X + Y)$	Distribute constants and signs
	$-X - (-2Y) - 3(4X) - 3(Y)$	Perform all multiplications
	$-X - 12X + 2Y - 3Y$	Group like terms
	$-13X - Y$	

Solving Linear Equations

The solution to an equation is a number that, when substituted into the equation for the variable, results in a true equation. Solving the equation means finding all such numbers. The general method of solving an equation is to replace the given equation with simpler and simpler equations until an equation of the form $variable = constant$ occurs. The constant is then called the solution of the equation.

The two properties that are used to form these simpler equations are *The Additive Property of Equations* and *The Multiplicative Property of Equations*.

The Additive Property of Equations: Adding (or subtracting) the same quantity to both sides of an equation will not change the solution set of the equation.

Example 1:	$X - 8 = 3$	
	$X - 8 + 8 = 3 + 8$	Add 8 to both sides
	$X = 11$	

The Multiplicative Property of Equations: Multiplying (or dividing) both sides of an equation by the same nonzero number will not change the solution set of the equation.

Example 2:	$5X = 27$	
	$\frac{5X}{5} = \frac{27}{5}$	Divide both sides by 5
	$X = \frac{27}{5}$	

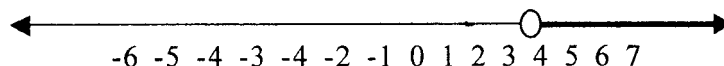
Anytime more than one property is used, always use the addition property before the multiplication property.

Example 3: $3X + 1 = 7$
 $3X + 1 - 1 = 7 - 1$
 $3X = 6$
 $\frac{3X}{3} = \frac{6}{3}$

Solving Inequalities

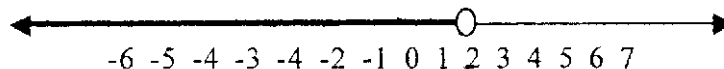
An inequality states that one quantity is larger than another. The solution set of an inequality is the set of all numbers that make the inequality true when substituted into the original inequality. These solution sets are found using similar properties as those used to solve equations.

Example: $Y + 3 > 7$
 $Y + 3 - 3 > 7 - 3$ Subtract 3 from both sides
 $Y > 4$ Graph the solution set

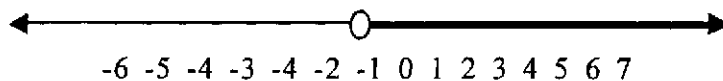


The Multiplicative Property of Inequalities: Multiplying (or dividing) both sides of an inequality by the same positive non-zero number will not change the solution set of the inequality. Multiplying (or dividing) both sides of an inequality by the same non-zero negative number *reversing the inequality* does not change the solution set of the inequality.

Example 2: $6X - 5 < 4X - 1$
 $6X - 4X - 5 < 4X - 4X - 1$ Subtract $4X$
 $2X - 5 + 5 < -1 + 5$ Add 5
 $\frac{2X}{2} < \frac{4}{2}$
 $X < 2$ Divide by positive 2



Example 3: $-3M + 4 \leq 7$
 $-3M + 4 - 4 \leq 7 - 4$ Subtract 4
 $-3M \leq 3$
 $\frac{-3M}{-3} \geq \frac{3}{-3}$
 $M \geq -1$



Exercises

Simplify each of the following

1. $-3V + 7V - V$

2. $12X - 3Y + 5X + 5Y$

3. $-7M + 9(2M - 3)$

4. $-(2X - 7Y) - 3(2Y - 11X)$

5. $3[2X - 3(X - 2Y)] + 3Y$

Solve each of the following Equations

6. $4Y + 7 = 5Y + 13$

7. $5(X - 2) = 6(X + 3)$

8. $\frac{Z}{20} + \frac{1}{12} = \frac{Z}{15} - \frac{1}{10}$

9. $\frac{4W + 23}{5} = \frac{19 - W}{7}$

10. $-5Y - 9(2Y + 3) = 7(3Y - 2) - (Y - 7) + 23$

11. Solve for X: $(3X + Y) - (Y - X) = 5(2X + Y)$

12. The sum of two numbers is 18. The difference between four times the smaller number and seven is equal to the sum of two times the larger number and five. Find the two numbers.

13. Jane bought a television set at a 20% off sale. If she paid \$360 for the set, what was the original price.

14. How many gallons of water must be mixed with 5 gal of 20% salt solution to make a 16% salt solution?

15. The perimeter of a triangle is 23ft. One side is twice the second side. The third side is 3ft. more than the second side. Find the measurements of each side.

Solve the following inequalities. Graph your results.

16. $2X < 6X - 8$

17. $13 - 4X < X - 2$

18. $4X + 9 > X - 7$

19. Two sides of a triangle must be 8m and 12m. The perimeter must be at least 24m at most 39m. Determine the length that can be used for the third side.

20. An insurance company is developing a new policy that is expected to have an average annual claim of at least \$409. The yearly premium for the policy will be \$390. To make up the underwriting loss the company plans to put the premium money into an investment that is expected to yield an annual profit of between 4.3% and 4.8% after other expenses are covered. Under these conditions, will this policy make money for the company?

EXPONENTS AND POLYNOMIALS

Integer Exponents

The following properties are used to work with exponents.

$X^A X^B = X^{A+B}$ To multiply with the same base you add exponents

$\frac{X^A}{X^B} = X^{A-B}$ To divide with the same base you subtract exponents

$(XY)^A = X^A Y^A$ Exponents distribute over multiplication

$(X^A)^B = X^{AB}$ A power to a power is the product of the powers

$X^{-A} = \frac{1}{X^A}$ Negative exponents imply reciprocals

Example 1: $\frac{(XY^5Z^3)(X^3Y^3Z)}{X^4Y^8Z^4}$ Add exponents for like bases

Example 2: $\frac{(2X^2)^3}{2^3X^6}$
 $8X^6$ Multiple Exponents

Example 3: $\frac{8X^{12}}{12X^9}$
 $\frac{8}{12} X^{12-9}$ Divide the numerical coefficients
Subtracts the exponents on the variables
 $\frac{2X^3}{3}$ Simplify

Example 4: $\frac{A^3B^{-2}}{A^{-5}B^4}$
 $\frac{A^3A^5}{B^2B^4}$ Make all exponents positive
 $\frac{A^8}{B^6}$ Add exponents for the like bases

Adding and Subtracting Polynomials

Simplifying polynomials is very similar to simplifying algebraic expressions. First remove all grouping symbols; then simplify each term and finally combine all like terms. Adding polynomials means adding the like terms together. To subtract polynomials, change the signs of the second polynomial and then add the polynomials.

Example 1:	Add	$(3X^3 - 7X + 2) + (7X^2 + 2X - 7)$ $(3X^3 + 7X^2 + (-7X + 2X) + (2 - 7))$ $3X^3 + 7X^2 - 5X - 5$	Use the properties to group like terms together
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Example 2:	Subtract	$(5X^2 + 3X - 6) - (3X^2 - 5X + 2)$ $(5X^2 + 3X - 6) + (-3X^2 + 5X - 2)$ $(5X^2 - 3X^2) + (3X + 5X) + (-6 - 2)$ $2X^2 + 8X - 8$	Change the signs and add Group like terms
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Multiply Polynomials

To multiply a monomial times a polynomial, use the distributive property.

Example 1:	$-3X^2(4X^2 - 5X + 11)$ $-3X^2(4X^2) - 3X^2(-5X) - 3X^2(11)$ $-12X^4 + 15X^3 - 33X^2$	Use the distributive property Simplify each term
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To multiply a polynomial times another polynomial, use the distributive property to multiply each term of the first polynomial by each term of the second polynomial.

Example 2:	$(X-2)(X^2 + 3X - 4)$ $X(X^2 + 3X - 4) - 2(X^2 + 3X - 4)$ $X(X^2) + X(3X) + X(-4) - 2(X^2) - 2(3X) - 2(-4)$ $X^3 + 3X^2 - 4X - 2X^2 - 6X + 8$ $X^3 + (3X^2 - 2X^2) + (-4X - 6X) + 8$ $X^3 + X^2 - 10X + 8$	Distributive property Distributive property again Simplify each term Group like terms Add like terms
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To multiply two binomials together use the shortcut called the **FOIL** method.

Example 3:	$(2X + 4)(X - 7) = 2X^2 - 14X + 4X - 28$ <div style="display: flex; justify-content: center; gap: 20px;"> F O I L </div>	First terms $(2X)(X)$ Outer terms $(2X)(-7)$ Inner terms $(4)(X)$ Last terms $(4)(-7)$ Add like terms
	$2X^2 - 10X - 28$	

Dividing Polynomials

To divide a polynomial by a monomial, divide each term of the numerator by the denominator.

Example:
$$\frac{12X^2Y - 6XY + 4X^2}{2XY}$$

$$\frac{12X^2Y}{2XY} - \frac{6XY}{2XY} + \frac{4X^2}{2XY}$$

$$6X - 3 + \frac{2X}{Y}$$

To divide one polynomial by another polynomial, use a method very similar to arithmetic long division.

Example:
$$\begin{array}{r} X-2 + \frac{2}{X-3} \\ X-3 \overline{) X^2 - 5X + 8} \\ \underline{X^2 - 2X} \\ -2X + 8 \\ \underline{-2X + 6} \\ 2 \end{array}$$

$$\frac{X^2}{X-3}$$

 Think $X = X$
 Multiply $(X - 3)(X) = X^2 - 3X$
 Subtract $(X^2 - 5X + 8) - (X^2 - 3X) = -2X + 8$
 Repeat the three step process again
 Remainder is written as a fraction

Exercises

Simplify each of the following

1. $(-2A^2B)^3$

2. $(2X^{-3}Y^4)(3X^4Y^2)$

3. $\frac{(2X^{-1}Y^2)(4X^2Y^{-3})^{-2}}{(12X^{-2}Y^{-2})^{-1}}$

4. $\frac{(10X^3Y^5)(21X^2Y^6)}{(7XY^3)(5X^9Y)}$

5. $(3X^3 - 2X^2 - 7) + (8X^2 - 8X + 7)$

6. $(-7X^2 + 3X - 6) - (8X^2 - 4X + 7) + (3X^2 - 2X - 1)$

7. $(-2X^3 + X^2 - 7)(2X - 3)$

8. $(2X - 7Y)(5X - 4Y)$

9. $(2X - 5)^2$

10. $(6X^3 + 10X^2 - 32) \div (3X - 4)$

11. A rectangle has a width of $X^2 + 7$ and a length of $X^2 + 5X + 4$. What is the perimeter of this rectangle? What is the area?

12. Subtract the square of $X - 6$ from the square of $X + 8$.

13. What polynomial when divided by $X - 7$, yields a quotient of $2X^2 + X - 6$?

14. The area of a rectangle is $4X^2 + 4X - 3$. Find the width if the length is $2X + 3$.

15. Two sides of a triangle are $3X^2 + 2X + 1$ and $4X^2 + 7$. Find the length of the third side if the perimeter is $12X^2 + 11X + 2$.

16. A piece of cardboard 18 inches wide by 24 inches long is to have square pieces cut out of each corner so that it can be made into a box. If the side of each square that will cut out is X inches by X inches, write a polynomial that represents the volume of the box.

FACTORING POLYNOMIALS

Using Common Factors

Factoring a polynomial means writing the polynomial as a product of two terms. To factor using common factors, first find the greatest common factor (GCF) of the terms of the polynomial; then divide the GCF into each term.

Example 1:	Factor	$\frac{4X^3Y^2 + 12X^2Y + 20XY^2}{4XY}$ $4XY(X^2Y + 3X + 5Y)$	GCF = 4XY Divide each term by 4XY Write answer as a product
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Example 2:	Factor	$3XZ - 4YZ - 3XA + 4YA$ $(3XZ - 4YZ) - (3XA - 4YA)$ $Z(3X - 4Y) - A(3X - 4Y)$ $(3X - 4Y)(Z - A)$	Group first two terms and Second two terms together Factor GCF from each group Factor (3X - 4Y) from each term
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Factoring Trinomials

Quadratic trinomials factor into the product of two binomials. Factoring trinomials is based on the FOIL method of multiplication. Note that the first term of the trinomial is the product of the first terms of the binomials. The last term of the trinomial is the product of the last terms of the binomials. The middle term of the trinomial is the sum of the products of the two inner terms and the two outer terms.

To factor a quadratic trinomial, first list all possible factors for the first term and the last term. Write out these choices as trial factors; then find the sum of the outer and inner products. Compare this sum to the middle term to determine the correct factorization.

Example 1: Factor $X^2 - 7X + 12$
 The last term has a positive sign; therefore both signs are alike. The sign on the middle term is negative; therefore both signs are negative. The factors of X^2 must be X and X. The factors of 12 are -1, -12 or -2, -6 or -3, -4.

<u>Trial factors</u>	<u>Sum outer and inner products</u>
$(X - 1)(X - 12)$	-13X
$(X - 2)(X - 6)$	8X
$(X - 3)(X - 4)$	-7X

Therefore the answer is $(X - 3)(X - 4)$

Example 2: Factor $4A^2 - A - 5$

The last term has a negative sign; therefore one sign will be positive, and the other will be negative. The factors of $4A^2$ are A and $4A$ or $2A$ and $2A$. The factors of -5 are -1 and 5 or 1 and -5 .

<u>Trial factors</u>	<u>Sum of outer and inner products</u>
$(A - 1)(4A + 5)$	A
$(A + 1)(4A - 5)$	$-A$
$(4A - 1)(A + 5)$	$19A$
$(4A + 1)(A - 5)$	$-19A$
$(2A - 1)(2A + 5)$	$8A$
$(2A + 1)(2A - 5)$	$-8A$

Therefore the answer is $(A + 1)(4A - 5)$

Simple Quadratic Equations

To solve quadratic equations use the **Principle of Zero Products**, which states that if the product of two factors is zero, then at least one of the factors must be zero.

Example:	Solve	$2X^2 - X = 1$	
		$2X^2 - X - 1 = 0$	Put into standard form
		$(2X + 1)(X - 1)$	Factor
		$2X + 1 = 0$ $X - 1 = 0$	Use the Principle of Zero Products
		$2X = -1$ $X = 1$	Solve each equation
		$X = \frac{-1}{2}$	

The solutions are $\frac{-1}{2}$ and 1 .

Exercises

Factor each of the following completely

- | | |
|----------------------|---------------------------|
| 1. $5X^3 - 35X^2$ | 2. $Z^2 + 9Z + 8$ |
| 3. $5A^2 - 180$ | 4. $11X^2 - 18X + 7$ |
| 5. $5N + M - MN - 5$ | 6. $6H^2K - 8HK^2 + 2K^3$ |

Solve each of the following questions

7. $3X^2 - 4X = 0$	8. $X^2 + 20 = 12X$
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9. Find three consecutive positive odd integers such that twice the product of the first two minus the product of the first and third is 49.

10. The length of a rectangle is 3ft. more than its width. Its area is 28 square ft. Find its dimensions.
11. A room contains 54 chairs. The number of chairs per row is three less than twice the number of rows. Find the number of rows and the number of chairs per row.
12. A strip of uniform width is to be cut off of both sides and both ends of a sheet of paper that is 8 inches by 11 inches in order to reduce the size of the paper to an area of 40 square inches. Find the width of the strip.
13. A brace wire is attached to the top of a tower 24m tall. It is anchored 7m from the base of the tower. How long is the piece of wire between these two points?
14. A ladder is leaning against a wall. The vertical distance up the wall to the top of the ladder is 4ft. less than the length of the ladder. The distance from the base of the ladder to the wall is 8ft. less than the length of the ladder. Find the length of the ladder.

RATIONAL EXPRESSIONS

Multiplying and Dividing Rational Expressions

Rational expressions are fractions where the numerator or the denominator or both are polynomials. The product of two rational expressions is a rational expression whose numerator is the product of the numerators and whose denominator is the product of the denominators.

Example: $\frac{X^2 - 2X}{2X^2 + X - 15} \cdot \frac{2X^2 - X - 10}{X^2 - 4}$

$\frac{X(X - 2)}{(X - 5)(X + 3)} \cdot \frac{(2X - 5)(X + 2)}{(X - 2)(X + 2)}$ Factor

$\frac{X(X - 2)(2X - 5)(X + 2)}{(2X - 5)(X + 3)(X - 2)(X + 2)}$ Multiply

$\frac{X \cancel{(X - 2)} \cancel{(2X - 5)} (X + 2)}{\cancel{(2X - 5)} (X + 3) \cancel{(X - 2)} (X + 2)}$ Simplify

$\frac{X}{X + 3}$ Write answer in simplest form

The quotient of two rational expressions is the product of the first rational expression and the reciprocal of the second rational expression.

Example: $\frac{6X^2 - 7X + 2}{3X^2 + X - 2} \div \frac{4X^2 - 8X + 3}{5X^2 + X - 4}$

$$\frac{6X^2 - 7X + 2}{3X^2 + X - 2} \quad \bullet \quad \frac{5X^2 + X - 4}{4X^2 - 8X + 3} \quad \text{Write as a multiplication problem}$$

$$\frac{(2X-1)(3X-2)}{(3X-2)(X+1)} \quad \bullet \quad \frac{(5X-4)(X+1)}{(2X-3)(2X-1)} \quad \text{Factor, Multiply, and Simplify}$$

$$\frac{5X-4}{2X-3} \quad \text{Write answer in simplest form}$$

Adding and Subtracting Rational Expressions

When adding or subtracting rational expressions with the same denominator, add or subtract the numerators and keep the common denominator. When adding or subtracting rational expressions with different denominators, first find the least common denominator for the expressions. Next express each rational expression in terms of the common denominator. Finally add or subtract the rational expressions and simplify the result.

Example: $\frac{Y+4}{Y^2-5Y+4} + \frac{Y+1}{Y-1}$

$$\frac{Y+4}{(Y-1)(Y+4)} + \frac{Y+1}{(Y-1)}$$

The denominators are factored

The least common denominator is $(Y-1)(Y-4)$

$$\frac{Y+4}{(Y-1)(Y-4)} + \frac{(Y+1)}{(Y-1)} \quad \bullet \quad \frac{(Y-4)}{(Y-4)}$$

Express each using the LCD

$$\frac{Y+4+(Y+1)(Y-4)}{(Y-1)(Y-4)}$$

Add the rational expressions

$$\frac{Y+4+Y^2-3Y-4}{(Y-1)(Y-4)}$$

Simplify

$$\frac{Y^2-2Y}{(Y-1)(Y-4)}$$

$$\frac{Y(Y-2)}{(Y-1)(Y-4)}$$

There are no common factors.

Complex Fractions

When both the numerator and the denominator of a complex fraction contain a single fraction, rewrite the complex fraction as a division problem and do the indicated division. If the numerator or the denominator contain more than one term, the terms must be combined before doing the division.

Example:
$$\frac{\frac{2X + 8}{4}}{\frac{X - X}{3}}$$

Working the numerator first

$$\begin{aligned} \frac{2X + 8}{4} &= \frac{2X}{4} + \frac{8}{1} \cdot \frac{4}{4} && \text{Rewrite using the LCD} \\ &= \frac{2X + 32}{4} && \text{Add} \\ &= \frac{2(X + 16)}{4} && \text{Simplify} \\ &= \frac{X + 16}{2} \end{aligned}$$

Working the denominator next

$$\begin{aligned} \frac{X - X}{3} &= \frac{X}{3} - \frac{X}{1} \cdot \frac{3}{3} && \text{Rewrite using the LCD} \\ &= \frac{X - 3X}{3} && \text{Subtract} \\ &= \frac{-2X}{3} && \text{Simplify} \end{aligned}$$

Thus

$$\frac{\frac{2X + 8}{4}}{\frac{X - X}{3}} = \frac{X + 16}{\frac{-2X}{3}}$$

$$\frac{X + 16}{2} \div \frac{-2X}{3} \quad \text{Write as a division problem}$$

$$\frac{X + 16}{2} \cdot \frac{3}{-2X} \quad \text{Write as equivalent multiplication problem}$$

$$\frac{-3(X + 16)}{4X} \quad \text{Multiply and Simplify}$$

Solving Equations with Rational Expressions

Equation containing rational expressions should first be cleared of all denominators by multiplying every term in the equation by the LCM of the denominators. The resulting linear or quadratic equation is then solved in the usual way. Always check all answers to eliminate any candidates that result in division by zero (excluded values).

$$\text{Example: } \frac{X}{X^2 - 36} + \frac{1}{X - 6} = \frac{4}{X + 6}$$

$$\frac{X}{(X - 6)(X + 6)} + \frac{1}{(X - 6)} = \frac{4}{(X + 6)} \quad \text{Factor the denominators}$$

The LCM of the denominators is $(X - 6)(X + 6)$ and then multiply by the LCM

$$\cancel{(X - 6)}(X + 6) \cdot \frac{X}{\cancel{(X - 6)}(X + 6)} + \cancel{(X - 6)}(X + 6) \cdot \frac{1}{\cancel{(X - 6)}} = \cancel{(X - 6)}\cancel{(X + 6)} \frac{4}{\cancel{X + 6}}$$

$$\begin{array}{l} X + X + 6 = 4(X - 6) \\ 2X + 6 = 4X - 24 \\ -2X = -30 \\ X = 15 \end{array} \quad \begin{array}{l} \text{Resulting Equation} \\ \text{Simplify} \end{array}$$

Since the excluded values are 6 and -6, 15 is the solution to the equation.

Exercises

Reduce each rational expression to lowest terms

1. $\frac{A^2 - 4}{A + 2}$

2. $\frac{9 - 16X^2}{16X^2 - 24X + 9}$

Perform the indicated operations

3. $\frac{3N - 6}{15N} \div \frac{N - 2}{20N^2}$

4. $\frac{A^2 - 9B^2}{A^2 - 6AB + 9B^2} \cdot \frac{A - 3B}{A + 3B}$

5. $\frac{Y}{X^2 - XY} + \frac{X}{Y^2 - XY}$

6. $\frac{X + 4}{X - 2} \cdot \frac{X - 6}{X^2 + X - 12} \div \frac{X + 5}{X^2 + 2X - 15}$

$$7. \frac{A - \frac{4}{A}}{A + \frac{4}{A}}$$

$$8. \frac{\frac{3}{X+3} + \frac{1}{X-1}}{\frac{1}{X-1} + \frac{2}{X+2}}$$

Solve each of the following equations.

$$9. \frac{4}{X+3} = \frac{2}{X}$$

$$10. \frac{5}{X^2 + X - 12} + \frac{7}{X^2 + 7X + 12} = \frac{21}{4X^2 - 36}$$

11. The speed of the current in a river is 5mph. If a boat can travel 198 miles with the current in the same time it could travel 138 miles against the current, what is the speed of the boat in still water?

12. An inlet pipe can fill a tank in 10 minutes. A drain can empty the tank in 12 minutes. If the tank is empty and both the pipe and the drain are open, how long will it take before the tank overflows?

13. Connie can type 600 words in 5 minutes less than it takes Katie to type 600 words. If Connie can type at a rate of 20 words per minute faster than Katie types, find Connie's typing rate.

14. The ratio of the squares to two consecutive even integers is 9/16. Find these integers.

ROOTS AND RADICALS

Simplifying Radical Expressions

Radical expressions are algebraic expressions which contain a radical. A radical expression is simplified when the radicand (expression under the radical) does not contain any powers greater than or equal to the index, there are no radicals in the denominator and all fractions are in the lowest terms.

Example: $\sqrt{162X^5Y^8}$

$$\sqrt{81 \cdot 2X^4XY^8}$$

Write factors as perfect squares if possible

$$9X^2Y^4 \sqrt{2X}$$

Remove the squares

Adding and Subtracting Radical Expressions

Adding and Subtracting can be done on terms with like radical portions. When the terms have like radical parts, add or subtract in the same manner as when adding or subtracting like terms.

Example: $\sqrt{45} + \sqrt{125} - \sqrt{75}$

$$\begin{aligned} &\sqrt{9 \cdot 5} + \sqrt{25 \cdot 5} - \sqrt{25 \cdot 3} \\ &3\sqrt{5 \cdot 5} + \sqrt{5 \cdot 5} - \sqrt{3} \\ &8\sqrt{5} - 5\sqrt{3} \end{aligned}$$

Write as product of squares

Remove squares

Combine like terms

Multiplying and Dividing Radical Expressions

Radical expressions are multiplied in the same way as other expressions. Coefficients are multiplied together and stay coefficients; radicands are multiplied together and remain radicands. The result is then simplified.

Example:

$$\begin{aligned} &\sqrt{6A^5B} \sqrt{8A^2B^2} \\ &\sqrt{48A^7B^3} && \text{Multiply} \\ &\sqrt{16 \cdot 3A^6AB^2B} && \text{Simplify} \\ &4A^3B\sqrt{3B} \end{aligned}$$

Example:

$$\begin{aligned} &(2\sqrt{5} - 3)(3\sqrt{5} + 4) \\ &6\sqrt{25} + 8\sqrt{5} - 9\sqrt{5} - 12 && \text{FOIL} \\ &6(5) - \sqrt{5} - 12 && \text{Simplify} \\ &30 - \sqrt{5} - 12 && \text{Combine like terms} \\ &18 - \sqrt{5} \end{aligned}$$

To divide one radical expression by another, write the division as a fraction and simplify.

Example:

$$\frac{\sqrt{21X^3} + \sqrt{27Y^5}}{\sqrt{3X}}$$

$$\frac{\sqrt{21X^3}}{\sqrt{3X}} + \frac{\sqrt{27Y^5}}{\sqrt{3X}} \quad \text{Write each division separately}$$

$$\sqrt{\frac{21X^3}{3X}} + \sqrt{\frac{27Y^5}{3X}}$$

Write as one radical

$$\sqrt{7X^2} + \sqrt{\frac{9Y^5X}{X^2}}$$

Simplify

$$X\sqrt{7} + \frac{3Y^2\sqrt{XY}}{X}$$

To simplify a fraction that has a binomial radical expression in the denominator, multiply the numerator and denominator by the conjugate form of the denominator and simplify.

Example: $\frac{2}{\sqrt{5} + 2}$

$$\frac{2}{\sqrt{5} + 2} \cdot \frac{\sqrt{5} - 2}{\sqrt{5} - 2}$$

Multiply by the conjugate

$$\frac{2(\sqrt{5} - 2)}{\sqrt{25} - 2\sqrt{5} + 2\sqrt{5} - 4}$$

$$\frac{2(\sqrt{5} - 2)}{5 - 4}$$

$$2(\sqrt{5} - 2)$$

Solving Equations with Radical Expressions

Equations that contain radicals can be solved by using the equality property of powers, which states that if two real numbers are equal, then any natural power of these numbers is also equal. To use this property, first isolate the radical on one side of the equation. Next raise both sides of the equation to the same power and solve as usual. Always check all answers to eliminate “imposter roots” that may be introduced by the squaring process.

Example: $\sqrt{X^3 + 3X - 17} + 2 = X$

$$\sqrt{X^2 + 3X - 17} = X - 2$$

Isolate the radical

$$(\sqrt{X^2 + 3X - 17})^2 = (X - 2)^2$$

Square both sides

$$X^2 + 3X - 17 = X^2 - 4X + 4$$

FOIL

$$3X - 17 = -4X + 4$$

Subtract X^2 from both sides

$$7X = 21$$

Combine like terms

$$X = 3$$

Divide by 7

Example: $\sqrt{32 + 3(3) - 17} + 2 = 3$

$$\sqrt{9 + 9 - 17} + 2 = 3$$

$$\sqrt{1} + 2 = 3$$

$$3 = 3$$

The root is true

Exercises

- $\sqrt{18A^3B^6}$
- $3\sqrt{108} - 2\sqrt{18} - 3\sqrt{48}$
- $5A\sqrt{3A^3B} + 2A^2\sqrt{27AB} - 4\sqrt{75A^5B}$
- $(\sqrt{2} - 3)(\sqrt{2} + 4)$
- $2\sqrt{3X^2} \cdot 3\sqrt{12XY^3} \cdot \sqrt{6X^3Y}$
- $\frac{9}{\sqrt{3A}}$
- $\frac{4 - 2\sqrt{5}}{2 - \sqrt{5}}$

Solve each of the following:

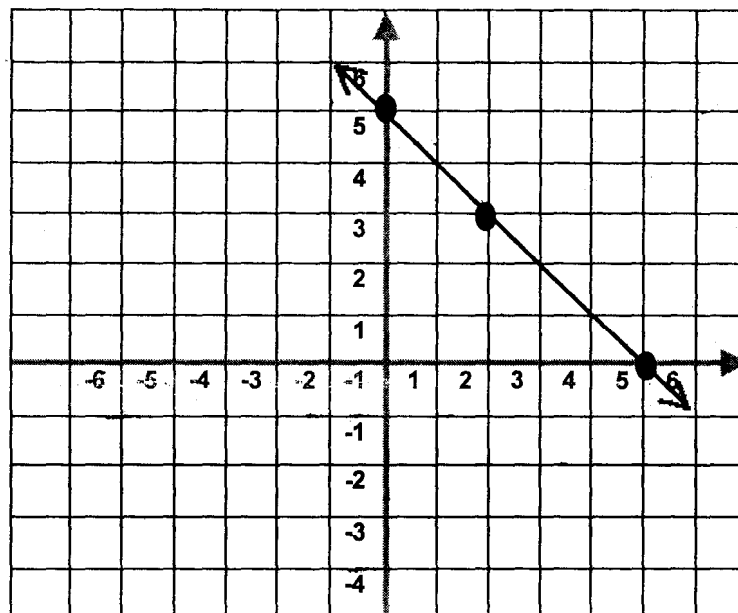
- $\sqrt{4X - 3} - 5 = 0$
- An object is dropped from an airplane. Find the distance the object has fallen when the speed reaches 400 feet per second. Use the equation $V = \sqrt{64d}$, where V is the speed of the object and d is the distance.
- How fare would a submarine periscope have to be above water to locate a ship 4.2 miles away? The equation of the distance in miles that the lookout can see is $d = \sqrt{1.5h}$, where the height in feet above the surface of the water and distance.
- Find the width of a rectangle that has a diagonal of 10 feet if the length is 2 feet longer than the width.
- When trying to put out a fire, fire fighters are interested in the flow rate of the water available at the site of the fire. The flow rate is determined by the equation $Q = 29.7 D^2 \sqrt{P}$ where Q is flow rate in gallons per minute, d is diameter of the hose nozzle in inches and P is static pressure in pounds per inch. What is the minimum static pressure to get at least 594 gal/min from a 2in fire hose?

GRAPHING

Graphing Linear Equations

The graph of a linear equation in two variables is the set of all ordered pairs which make the equation true. These ordered pairs can be obtained by arbitrarily assigning a value to one member of the ordered pair (usually the X) and then calculating the corresponding other member (Y). To graph a linear equation find at least three solution pairs in this manner. Plot these points on the coordinate plane. Finally draw the straight line which passes through these three points.

Example: Graph $X + Y = 5$
Choose $X = 0$, $0 + Y = 5$, $Y = 5$ (0, 5) is a point
Choose $X = 2$, $2 + Y = 5$, $Y = 3$ (2, 3) is a point
Choose $X = 5$, $5 + Y = 5$, $Y = 0$ (5, 0) is a point

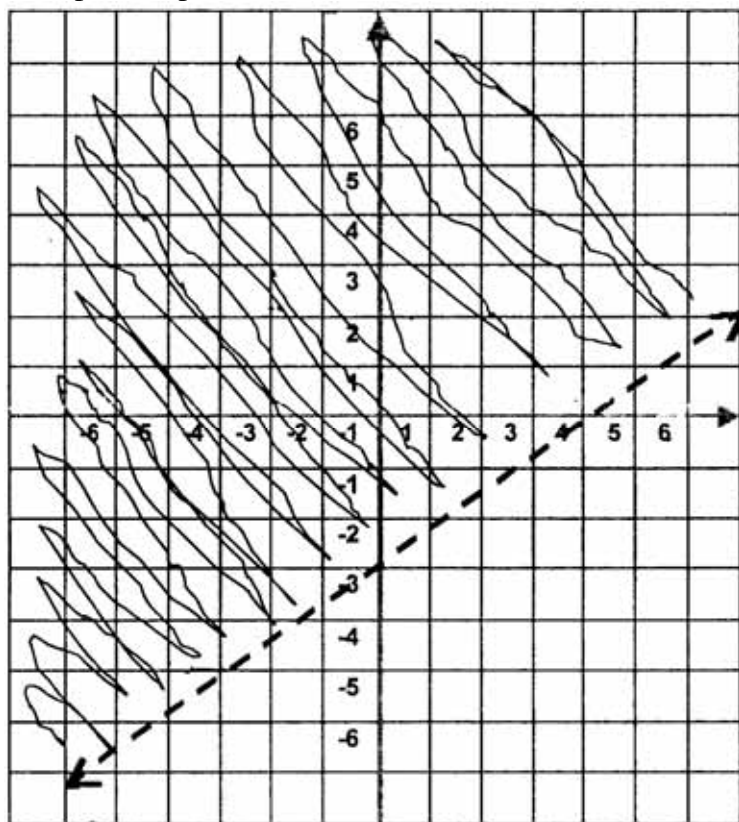


The plotting of these points may not be exact due to software limitations.

Graphing Linear Inequalities

The graph of a linear inequality is a region of the coordinate plane called a half-plane. The graph of the related linear equality forms the boundary line between the half-plane that is the solution set and the half-plane that is not. To form the graph, first plot the related equation. This boundary line is dashed if the inequality is a strict inequality ($<$ or $>$) and solid if the inequality contains \leq or \geq . The solution set is then determined by selecting any point not on the boundary and testing this point for truthfulness. (*See next page*)

Example: Graph $3X - 4Y < 12$



The plotting of these points may not be exact due to software limitations.

1. Graph the related line.
The line is plotted dashed.
2. Choose the point $(0, 0)$ and check these values in the original inequality.
 $3(0) - 4(0) < 12$
 $0 < 12.$

Note: If $(0, 0)$ is on the boundary, select another point such as $(0, 1)$ or $(1, 0)$ not on the boundary.

3. Shade the half-plane containing $(0, 0)$

Note: If the point tested produces a false statement, shade the half-plane that does not contain the tested point.

Slope of a Line

Slope is a measure of the steepness of a line. Slope is defined by the formula:

$$\text{Slop} = m = \frac{\text{change in } Y}{\text{change in } X}$$

Slope is calculated using the formula:

$$m = \frac{Y_2 - Y_1}{X_2 - X_1}$$

When (X_1, Y_1) are the coordinates from point 1 and (X_2, Y_2) are the coordinates from the point 2. It should be noted that it does not matter in which order the two points are considered. The labeling of point 1 and point 2 is arbitrary, as long as both coordinates represent the point labeled as point 1 are used as X_1 and Y_1 and both coordinates representing the point labeled as point 2 are used as X_2 and Y_2 . Furthermore it does not matter which two points on any given line are used to determine the slope, the slope remains the same regardless of the point chosen to compute it.

Example: Find the slope of the line through (-2, -3) and (2, 5)

$$m = \frac{5 - (-3)}{2 - (-2)} = \frac{8}{4} = \frac{2}{1}$$

Thus the slope is 2.

Parallel lines are two distinct lines which do not intersect and have the same slope.

Perpendicular lines are two distinct lines which intersect at right angles. In perpendicular lines, the product of the slopes of the line is -1. The slopes of perpendicular lines are opposite reciprocals.

Writing Equations of Lines

The slope intercept form of the line is used in writing the equation of a line. First the slope is determined from the given information. The y- intercept is then determined by using this slope and any given point. From this information the equation of the line is written.

Example: Write the equation of the line through (5, 1) and (-3, -7)

$$m = \frac{-7 - 1}{-3 - 5} = \frac{-8}{-8} = 1 \quad \text{Determine the slope}$$

$$Y = mX + b \quad \text{Slope intercept form of line}$$

$$1 = 1(5) + b \quad \text{Substitute } m = 1 \text{ and point } (5, 1)$$

$$-4 = b \quad \text{Solve for y-intercept}$$

$$Y = X - 4 \quad \text{Equation of a line}$$

Example: Write the equation of a line parallel to $Y = 6X - 1$ and through the point (-1, -2)

$$M = 6 \quad \text{Read slope from line}$$

$$Y = mX + b \quad \text{Slope intercept form of the line}$$

$$-2 = 6(-1) + b \quad \text{Substitute } m = 6 \text{ and point } (-1, -2)$$

$$4 = b \quad \text{Solve for } b$$

$$Y = 6X + 4 \quad \text{Equation of the line}$$

Exercises

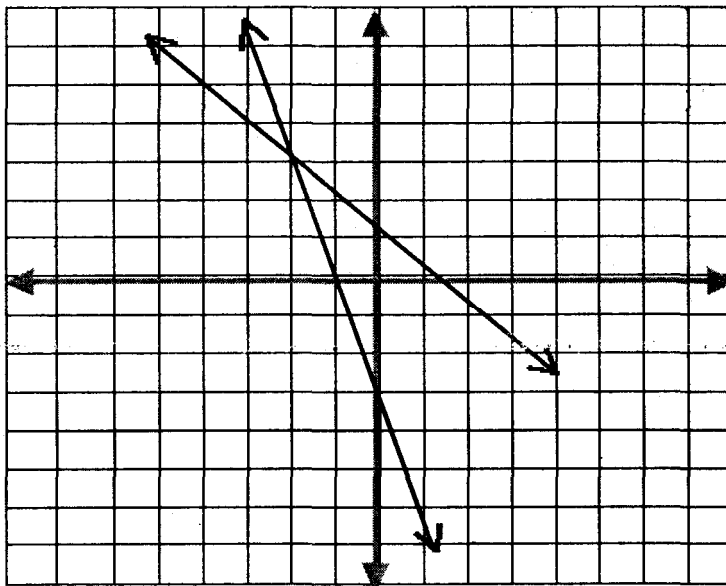
1. Graph $2X - 3Y = 6$
2. Graph $Y - X < 3$
3. Find the slope of the line that passes through (6, 0) and (-5, 4).
4. Determine whether $\frac{1}{2}x - 4 = Y$ and $X + 9 = 2Y$ are parallel, perpendicular, collinear, or intersecting.
5. Find the equation of the line that contains the point (2, 4) and is perpendicular to $Y = \frac{2}{3}X + 2$.
6. The cost for a buffet special at the Skylite Diner is a “plate fee” plus an amount per ounce. The cashier writes the number of ounces and price owed as an ordered pair. She writes (10, 4.85) and (22, 8.33) for two orders. Write the equation of a line; then determine the “plate fee” and cost per ounce.
7. The cost of producing plastic tubs is linearly related to the number of tubs produced. The production cost is \$178 for 28 tubs in March and \$193 for 33 tubs in April. How much is the variable cost of production per tub? How much is the overhead (cost of operation even if no tubs are produced)?
8. A manufacturer determines that the relationship between profit earned P , and the number of items sold X , is linear. Suppose profit is \$1500 for 45 items and \$2500 on 65 items. What would be the expected profit on 100 items?
9. A three story building is on fire. When the fire fighters arrive at the scene, it is determined (for safety purposes) that all fire fighters must be at least 70 feet from the fire. The maximum horizontal distance of a stream of water is determined by the equation $S = .05N + 40D - 4$, where S is maximum horizontal distance, N is nozzle pressure in lb/sq.in. and D is diameter of the hose nozzle in inches. If the fire engine can only generate 48 lbs/sq.in. of nozzle pressure, what is the minimum nozzle diameter that will enable a stream of water to reach the edge of the fire?

SYSTEMS OF EQUATIONS

Solving Systems of Equations by Graphing

A system of equations is two or more equations considered together. A system in two variables can be solved by graphing the two lines on the same coordinate axes. The point of intersection is then the solution of the system. Always check the apparent ordered pair solution in both equations. (See next page.)

Example: $X + 2Y = 4$
 $2X + Y = -1$



The plotting of these points may not be exact due to software limitations.

Check: $-2 + 2(3) = 4$
 $2(-2) + 3 = -1$

Thus $X = -2$ and $Y = 3$ is the solution to this system.

If the lines are parallel, there is no solution to the system. These systems are called inconsistent. If both equations produce the same line, there are infinitely many solutions to the system. These systems are called dependent.

Solving Systems of Equations by Elimination

A common algebraic method of solving systems of equations is the Elimination Method. In this method, each equation is first multiplied by a non-zero number so that the sum of the coefficients on either X or Y is zero. The equations are then added together producing a new equation in one variable. This equation is solved, and the solution substituted back into one of the original equations to obtain the solution value of the other variable.

Example: $2X - 5Y = 1$
 $3X - 4Y = 5$

Multiply the first equation by -3 and the second by 2 so that the sum of the coefficients on the X variable will be zero.

$$\begin{array}{r}
 -3[2X - 5Y = 1] \\
 2[3X - 4Y = 5] \\
 \hline
 -6X + 15Y = -3 \\
 6X - 8Y = 10 \\
 \hline
 7Y = 7 \\
 Y = 1
 \end{array}
 \quad
 \begin{array}{l}
 \text{Add equations} \\
 \text{Divide by 7}
 \end{array}$$

Substitute $Y = 1$ into the first equation and solve for X .

$2X - 5Y = 1$	First equation
$2X - 5(1) = 1$	Substitute for Y and Multiply
$2X - 5 = 1$	Add 5 to both sides
$2X = 6$	Divide by 2
$X = 3$	

Check: $2(3) - 5(1) = 1$
 $3(3) - 4(1) = 5$

Thus the solution to this system is $X = 3$ and $Y = 1$

Solving Systems of Equations by Substitution

Another common algebraic method for solving systems of equations is the Substitution Method. In this method one equation is solved for one of the variables. This algebraic expression is then substituted into the other equation, producing a new equation in a single variable. This new equation is solved and the resulting value substituted back into an original equation to solve for the other variable

Example: $Y = 3X - 7$
 $4Y - 3X = 8$

Since the first equation is already in the form solving for Y , $3X - 7$ will be substituted for the Y in the second equation.

$4Y - 3X = 8$	Second equation
$4(3X - 7) = 8$	Substitute $3X - 7$ for Y
$12X - 28 - 3X = 8$	Multiply & combine like terms
$9X - 28 = 8$	Add 28 to both sides
$9X = 36$	Divide by 9
$X = 4$	

Now that a value for X has been found, substitute this value back into the first original equation to obtain a value for the Y variable.

$Y = 3X - 7$	First equation
$Y = 3(4) - 7$	Substitute 4 for X
$Y = 12 - 7$	Multiply combine like terms
$Y = 5$	

Check: $5 = 3(4) - 7$
 $4(5) - 3(4) = 8$

Thus the solution to this system is $X = 4$ and $Y = 5$.

Exercises

1. Solve by graphing

$$X + Y = 3$$

$$3X - 2Y = -6$$

2. Solve by Addition Method

$$3X + 4Y = -2$$

$$2X + 5Y = 1$$

3. Solve by Substitution

$$5X + 2Y = 1$$

$$2X + 3Y = 7$$

4. A motorboat traveling with the current went 36 miles in two hours. Against the current it took an hour longer to go the same distance. Find the rate of the boat in calm water and the rate of the current.

5. A coin bank contains only nickels and dimes. The total value of the coins is \$2.50. If the nickels were dimes and the dimes were nickels, the value of the coins would be \$3.50. How many nickels are in the bank?

6. A chemist has two alloys, one of which is 10% gold and 15% lead, the other of which is 30% gold and 40% lead. How many grams of each of these two alloys should be used to make an alloy that contains 60 grams of gold and 88 grams of lead?

7. How much water should be evaporated from 75 ounces of a 2% saline solution to produce a 5% solution?

8. The difference between the ages of an oil painting and a watercolor is 35 years. The age of the oil painting five years from now will be twice the age of the watercolor five years ago. Find the age of each painting.

9. Two people, one rollerblader and one race walker, are 18 miles apart. They will meet in two hours if they head toward each other. They will meet in four hours if they head in the same direction – that is – if the rollerblader heads toward the walker. Find the rate of each.

ANSWERS

Equations and Inequalities

1. $3V$	11. $-5Y/6$
2. $17X + 2Y$	12. 8 AND 10
3. $11M - 27$	13. \$450
4. $31X + Y$	14. 1.25gal
5. $-3X + 21Y$	15. 5ft; 8ft; 10ft
6. -6	16. $X > 2$
7. -28	17. $X > 3$
8. 11	18. $X > -16/3$
9. -2	19. $4 < XC19$
10. -1	20. No

Exponents and Polynomials

1. $-8AB$	10. $2X^2 + 6X + 8$
2. $6XY^6$	11. perimeter: $4X^2 + 10x \pm 22$, area: $X^4 + 5X^3 + 11X^2 + 35X + 28$
3. $3Y^6$ $\frac{\quad}{2X^7}$	12. $28X + 28$
4. $\frac{6Y^7}{X^5}$	13. $2X^3 - 13X^2 - 13X + 42$
5. $3X^3 + 6X^2 - 8X$	14. $2X - 1$
6. $-12X^2 + 5X - 14$	15. $5X^2 + 9X - 6$
7. $-4X^4 + 8X^3 - 3X^2 - 14X + 21$	16. $4X^3 - 84X^2 + 432X$
8. $10X^2 - 43XY + 28Y^2$	
9. $4X^2 - 20X + 25$	

Factoring Polynomials

1. $5X^2(X - 7)$	8. 10, 2
2. $(Z + 1)(Z + 8)$	9. 7, 9, 11
3. $5(A + 6)(A - 6)$	10. 4ft by 7ft
4. $(11X - 7)(X - 1)$	11. 6 rows, 9 chairs per row
5. $(5 - M)(N - 1)$	12. 1.5in
6. $2K(3H - K)(H - K)$	13. 25 meters
7. 0, $\frac{4}{3}$	14. 20ft.

Rational Expressions

1. $A - 2$	9. 3
2. $\frac{-3 - 4X}{4X - 3}$	10. 4
3. $4N$	11. 28mph
4. 1	12. 60min
5. $\frac{-X - Y}{XY}$	13. 60 words per minute
6. $\frac{X - 6}{X - 2}$	14. 6 and 8
7. $\frac{A^2 - 4}{A^2 + 4}$	
8. $\frac{4(X + 2)}{3(X + 3)}$	

Roots and Radicals

1. $3AB^3\sqrt{2A}$	7. 2
2. $6\sqrt{3} - 6\sqrt{2}$	8. 7
3. $-9A^2\sqrt{3AB}$	9. 2500ft.
4. $-10 + \sqrt{2}$	10. 11.76ft.
5. $36X^3Y^2\sqrt{6}$	11. 6ft.
6. $\frac{3\sqrt{3A}}{A}$	12. 25 lbs/sq in

Graphing

<p>1.</p> <p>The points should be plotted at (0, -2) & (3, 0).</p>	<p>2.</p> <p>The points should be plotted at (0, 3) & (-3, 0).</p>
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3. $-\frac{4}{11}$

4. parallel

5. $Y = \frac{-3X + 7}{2}$

6. \$1.95, \$0.29/oz.

7. \$94, \$3/tub

8. \$4250

9. 1.25"

Systems of Equations

1. (0,3)
2. (-2, 1)
3. (-1,3)
4. boat 15mph, current 3mph
5. 30 nickels
6. first alloy 480g, second alloy 40g
7. 45oz.
8. Oil 85yrs, watercolor 50yrs
9. rollerblader 6.75mph, walker 2.25mph